

Math 601 – Spring 2014  
Handout #1

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1. Show that the inverse limit of the diagram:

$G$

$H$

is the direct product  $G \times H$ .

2. In the category of groups, consider the diagram:

$$G \begin{array}{c} \xrightarrow{f} \\ \xrightarrow{g} \end{array} H$$

where  $g$  is the trivial group homomorphism. Does the direct limit of this diagram necessarily exist? If so, describe the direct limit.

3. A **topological group** is a topological space and group  $G$  such that the group operations

$$\begin{array}{ccc} G \times G & \longrightarrow & G & \text{and} & G & \longrightarrow & G \\ (x, y) & \longmapsto & xy & & x & \longmapsto & x^{-1} \end{array}$$

are continuous. For such a space, prove that  $\pi_1(G, 1)$  is abelian.

4. Compute  $\pi_1(\mathbb{R}P^k)$ .
5. (Hatcher, number 1.2.11) The **mapping torus**  $T_f$  of a map  $f : X \rightarrow X$  is the quotient of  $X \times I$  obtained by identifying  $(x, 0)$  with  $(f(x), 1)$ . In the case  $X = S^1 \vee S^1$  with base-point preserving  $f$ , compute a presentation for  $\pi_1(T_f)$  in terms of the induced map  $f_* : \pi_1(X) \rightarrow \pi_1(X)$ . Do the same when  $X = S^1 \times S^1$ .
6. (Hatcher, number 1.2.8) Compute the fundamental group of the space obtained from two tori  $S^1 \times S^1$  by identifying a circle  $S^1 \times x_0$  in one torus with the corresponding circle  $S^1 \times x_0$  in the other torus.
7. (Hatcher, number 1.1.5) Show that for a space  $X$ , the following three conditions are equivalent:
- (a) Every map  $S^1 \rightarrow X$  is homotopic to a constant map, with image a point.
  - (b) Every map  $S^1 \rightarrow X$  extends to a map  $\mathbb{D}^2 \rightarrow X$ .
  - (c)  $\pi_1(X, x_0) = 1$  for all  $x_0 \in X$ .

Deduce that a space  $X$  is simply-connected iff all maps  $S^1 \rightarrow X$  are homotopic (in this problem, ‘homotopic’ means ‘homotopic without regard to basepoints’).

8. Use the Wirtinger presentation of the fundamental group of a knot complement (see Hatcher, number 1.2.22) to show that the unknot, the trefoil, and the figure-eight knot are all pairwise non-isotopic. These knots are shown in order on the diagram on the next page.

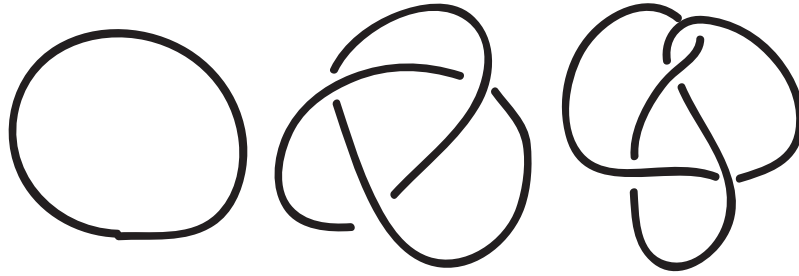


Figure 1: the unknot, the (right-handed) trefoil, and the figure-eight knot