Math 601 – **Spring** 2014
Handout #1

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1. Show that the inverse limit of the diagram:

$$\begin{matrix} G \\ \\ H \end{matrix}$$

   is the direct product $G \times H$.

2. In the category of groups, consider the diagram:

$$G \underset{g}{\overset{f}{\rightrightarrows}} H$$

   where g is the trivial group homomorphism. Does the direct limit of this diagram necessarily exist? If so, describe the direct limit.

3. A **topological group** is a topological space and group G such that the group operations

$$\begin{aligned} G \times G &\longrightarrow G \\ (x, y) &\longmapsto xy \end{aligned} \quad \text{and} \quad \begin{aligned} G &\longrightarrow G \\ x &\longmapsto x^{-1} \end{aligned}$$

   are continuous. For such a space, prove that $\pi_1(G, 1)$ is abelian.

4. Compute $\pi_1(\mathbb{RP}^k)$.

5. (Hatcher, number 1.2.11) The **mapping torus** T_f of a map $f : X \to X$ is the quotient of $X \times I$ obtained by identifying $(x, 0)$ with $(f(x), 1)$. In the case $X = S^1 \vee S^1$ with base-point preserving f, compute a presentation for $\pi_1(T_f)$ in terms of the induced map $f_* : \pi_1(X) \to \pi_1(X)$. Do the same when $X = S^1 \times S^1$.

6. (Hatcher, number 1.2.8) Compute the fundamental group of the space obtained from two tori $S^1 \times S^1$ by identifying a circle $S^1 \times x_0$ in one torus with the corresponding circle $S^1 \times x_0$ in the other torus.

7. (Hatcher, number 1.1.5) Show that for a space X, the following three conditions are equivalent:

   (a) Every map $S^1 \to X$ is homotopic to a constant map, with image a point.

   (b) Every map $S^1 \to X$ extends to a map $\mathbb{D}^2 \to X$.

   (c) $\pi_1(X, x_0) = 1$ for all $x_0 \in X$.

   Deduce that a space X is simply-connected iff all maps $S^1 \to X$ are homotopic (in this problem, 'homotopic' means 'homotopic without regard to basepoints').

8. Use the Wirtinger presentation of the fundamental group of a knot complement (see Hatcher, number 1.2.22) to show that the unknot, the trefoil, and the figure-eight know are all pairwise non-isotopic. These knots are shown in order on the diagram on the next page.

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Figure 1: the unknot, the (right-handed) trefoil, and the figure-eight knot